Station 5 Lösung
Wendepunkte

Gegeben sind die erste, zweite und dritte Ableitung einer Funktion inkl. der Nullstellen (x_i) der zweiten Ableitung. Bestimme die Wendepunkte der Ausgangsfunktion.

(1)

$$f(x) = \frac{1}{2}x^2 - e^{-x} \quad f'(x) = x + e^{-x} \quad f''(x) = 1 - e^{-x} \quad f'''(x) = -e^{-x}$$

$$x_0 = 0$$

$$f'''(0) = -e^{-0} = -1 \neq 0 \Rightarrow bei\ x = 0\ ist\ ein\ WP$$

$$f(0) = \frac{1}{2}0^2 - e^{-0} = -1 \quad WP(0/-1)$$

(2)

$$f(x) = x^2 e^{-x} \quad f'(x) = e^{-x}(2x - x^2) \quad f''(x) = e^{-x}(2 - 4x + x^2)$$

$$f'''(x) = e^{-x}(-6 + 6x - x^2) \quad x_0 = 2 + \sqrt{2} \quad x_1 = 2 - \sqrt{2}$$

$$f'''(2+\sqrt{2}) = (2+\sqrt{2})^2 e^{-(2+\sqrt{2})} \neq 0 \Rightarrow bei\ x = 2+\sqrt{2}\ ist\ ein\ WP$$

$$f(2+\sqrt{2}) = (2+\sqrt{2})^2 e^{-(2+\sqrt{2})} \quad WP(2+\sqrt{2}\,/(6+4\sqrt{2})e^{-(2+\sqrt{2})})$$

$$f'''(2-\sqrt{2}) = (2-\sqrt{2})^2 e^{-(2-\sqrt{2})} \neq 0 \Rightarrow bei\ x = 2-\sqrt{2}\ ist\ ein\ WP$$

$$f(2-\sqrt{2}) = (2-\sqrt{2})^2 e^{-(2-\sqrt{2})} \quad WP(2-\sqrt{2}\,/(6-4\sqrt{2})e^{-(2-\sqrt{2})})$$

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(3)

$$f(x) = xe^{-kx^2} \quad f'(x) = e^{-kx^2}(1-2kx^2) \quad f''(x) = e^{-kx^2}(-6kx+4k^2x^3)$$

$$f'''(x) = e^{-kx^2}(-6k+24\,k^2x^2-8\,k^3x^4)$$

$$x_0 = 0 \quad x_1 = \left(\sqrt{\frac{3}{2k}}\right) \quad x_2 = -\sqrt{\frac{3}{2k}} \quad k>0$$

$$f'''(0) = e^{-k0^2}(-6k+24\,k^2\,0^2-8\,k^3\,0^4) = (-6k) \neq 0 \Rightarrow bei\ x=0\ ist\ ein\ WP$$

$$f(0) = 0e^{-k0^2} = 0 \quad WP(0/0)$$

$$f'''\left(\sqrt{\frac{3}{2k}}\right) = e^{-k\left(\sqrt{\frac{3}{2k}}\right)^2}(-6k+24\,k^2\left(\sqrt{\frac{3}{2k}}\right)^2-8\,k^3\left(\sqrt{\frac{3}{2k}}\right)^4)$$

$$= e^{-\frac{3}{2k}}(-6k+36\,k-18\,k) = e^{-\frac{3}{2k}}\cdot 12\,k \neq 0$$

$$\Rightarrow bei\ x = \left(\sqrt{\frac{3}{2k}}\right) ist\ ein\ WP$$

$$f\left(\sqrt{\frac{3}{2k}}\right) = \left(\sqrt{\frac{3}{2k}}\right)e^{-k\left(\sqrt{\frac{3}{2k}}\right)^2} \quad also\ \ WP\left(\sqrt{\frac{3}{2k}}/\left(\sqrt{\frac{3}{2k}}\right)e^{-\frac{3}{2k}}\right)$$

$$f'''\left(-\sqrt{\frac{3}{2k}}\right) = e^{-k\left(-\sqrt{\frac{3}{2k}}\right)^2}(-6k+24\,k^2\left(-\sqrt{\frac{3}{2k}}\right)^2-8\,k^3\left(-\sqrt{\frac{3}{2k}}\right)^4)$$

$$= e^{-\frac{3}{2k}}(-6k+36\,k-18\,k) = e^{-\frac{3}{2k}}\cdot 12\,k \neq 0$$

$$\Rightarrow bei\ x = -\left(\sqrt{\frac{3}{2k}}\right) ist\ ein\ WP$$

$$f\left(-\sqrt{\frac{3}{2k}}\right) = \left(-\sqrt{\frac{3}{2k}}\right)e^{-k\left(-\sqrt{\frac{3}{2k}}\right)^2} \quad also\ \ WP\left(-\sqrt{\frac{3}{2k}}/\left(-\sqrt{\frac{3}{2k}}\right)e^{-\frac{3}{2k}}\right)$$