Station 6 Lösung
Tangente/Normale

Bestimme die Tangenten- und Normalengleichung an der angegebenen Stelle. Zur Erinnerung: Steigung der Normalen $m_n = -\frac{1}{m_t}$, allgemeine Geradengleichung $y = mx + b$.

(1)

$$f(x) = x + e^{-x} \quad f'(x) = 1 - e^{-x} \quad x = \frac{1}{2}$$

$$m_t = f'\left(\frac{1}{2}\right) = 1 - e^{-\left(\frac{1}{2}\right)} \approx 0{,}393 \quad \Rightarrow m_n = -\frac{1}{0{,}393} \approx -2{,}541$$

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right) + e^{-\left(\frac{1}{2}\right)} \approx 1{,}107$$

$$t\left(\frac{1}{2}\right) = 0{,}393 \cdot \left(\frac{1}{2}\right) + b = 1{,}107 \Leftrightarrow b = 0{,}9105$$

$$\Rightarrow t_{\frac{1}{2}}(x) = 0{,}393x + 0{,}9105$$

$$n\left(\frac{1}{2}\right) = -2{,}541 \cdot \left(\frac{1}{2}\right) + b = 1{,}107 \Leftrightarrow b = 2{,}3775$$

$$\Rightarrow n_{\frac{1}{2}}(x) = -2{,}541x + 2{,}3775$$

(2)

$$f(x) = x^2 e^{-x} \quad f'(x) = e^{-x}(2x - x^2) \quad x = 1$$

$$m_t = f'(1) = e^{-1}(2-1) \approx 0{,}368 \quad \Rightarrow m_n = -\frac{1}{0{,}368} \approx -2{,}717$$

$$f(1) = 1^2 e^{-1} \approx 0{,}368$$

$$t(1) = 0{,}368 \cdot 1 + b = 0{,}368 \Leftrightarrow b = 0$$

$$\Rightarrow t_1(x) = 0{,}368x$$

$$n(1) = -2{,}717 \cdot 1 + b = 0{,}368 \Leftrightarrow b = 3{,}085$$

$$\Rightarrow n_1(x) = -2{,}717x + 3{,}085$$

(3)

$$f(x) = x^2 e^{-x} \quad f'(x) = e^{-x}(2x - x^2) \quad x = 2$$

$$m_t = f'(1) = e^{-2}(4-4) = 0 \quad \Rightarrow m_n = -\frac{1}{0} \; \textit{existiert nicht!}$$

$$f(1) = 2^2 e^{-2} \approx 0{,}541$$

$$t(1) = 0 \cdot 2 + b = 0{,}541 \Leftrightarrow b = 0{,}541$$

$$\Rightarrow t_2(x) = 0{,}541$$

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(4)

$$f_k(x) = x e^{-k x^2} \qquad f_k'(x) = e^{-k x^2}(1 - 2 k x^2) \quad k = 2, \quad x = 2$$

$$m_t = f_2'(2) = e^{-2 \cdot 2^2}(2 - 4 \cdot 2^2) \approx -0{,}005 \quad \Rightarrow m_n = -\frac{1}{-0{,}005} \approx 200$$

$$f_2(2) = 2 e^{-2 \cdot 2^2} \approx 0{,}00067$$

$$t(1) = -0{,}0005 \cdot 2 + b = 0{,}00067 \Leftrightarrow b = 0{,}00167$$

$$\Rightarrow t_2(x) = -0{,}0005 x + 0{,}00167$$

$$n(1) = 200 \cdot 2 + b = 0{,}00067 \Leftrightarrow b = 399{,}99$$

$$\Rightarrow n_2(x) = 200 x + 399{,}99$$